Advanced Signal Processing Methods for Analysis of Non-Stationary Signals in Power Systems

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Abstract — This paper aims to consider using the Wavelet Transform (WT), Wigner-Ville Distribution (WVD) and Choi-Williams Distribution (CWD) applied for spectrum estimation of non-stationary signals in power systems. The main goal is to emphasize advantages and disadvantages of mentioned methods in comparison with Short-Time Fourier Transform (STFT). Especially, the aspect of frequency resolution are explored on the basis of signals at the output of simulated frequency converter, which is one of the major issues when diagnosing problems of some drive faults are considered.

Keywords — Fault diagnosis, power systems, wavelet transform, Wigner-Ville representation.

I. INTRODUCTION

The aim of signal analysis is to extract relevant information from a signal by transforming it. Spectrum estimation of discretely sampled deterministic and stochastic processes is usually based on procedures employing the Fast Fourier Transform (FFT). Conventional FFT spectral estimation is based on a Fourier series model of the data, that is the process is assumed to be composed of a set of harmonically related sinusoids. This approach to spectrum analysis is computationally efficient and produces reasonable results for a large class of signal processes. In spite of these advantages there are several inherent performance limitations of the FFT approach. The most prominent limitation is that of frequency resolution, i.e. the ability to distinguish the spectral responses of two or more signals. Because of some invalid assumptions (zero data or repetitive data outside the duration of observation) made in this method, the estimated spectrum can be a smeared version of the true spectrum. A second limitation is due to windowing of the data. Windowing manifests itself as leakage in the spectral domain — energy in the main lobe of a spectral response leaks into the side-lobes, obscuring and distorting other spectral responses that are present. These two performance limitations of the FFT approach are especially troublesome when analyzing short data records. Short data records occur frequently in practice, because many measured processes are brief in duration or have slowly time-varying spectra, that can be considered constant only for short record lengths. Limitations of the FFT approach, many alternative spectral estimation procedures have been proposed within the last 4-5 decades.

In the case of a non-stationary signal, any change of the signal causes a continuous spectrum which spread out over the whole frequency axis [5]. Therefore other methods of analysis are needed, to get a two-dimensional time-frequency representation $S(t,\omega)$ of the investigated signal. First, Gabor has adapted the Fourier Transform to define the $S(t,\omega)$, assuming that the signal is stationary when seen through a window of limited extent. This yields the Short-Time Fourier Transform (STFT). The time varying spectra of non-stationary time series commonly used are spectrograms, from the STFT. If a signal is composed of small bursts of components, then each type of component can be analyzed with good time resolution or frequency resolution, but not both [10]. To overcome the resolution limitation, the Wavelet Transform (WT) has been developed. [9]. Wavelet Transform provides a unified framework for a number of methods, which have been developed independently for various signal processing applications. In contrast to the STFT, the WT uses short windows at high frequencies and long windows for low frequencies. Using the WT, the time-varying spectra of non-stationary signals can also be obtained in form of scalograms. Scalogram is defined as the squared modulus of the WT. In contrast to the spectrogram information about the signal is here shown with different resolutions.

Time-varying spectra presented as spectrograms can be also obtained using the Wigner-Ville Distribution (WVD). The Wigner-Ville spectrum shows better frequency concentration and less phase dependence than Fourier spectra. However, for multicomponent signal $x(t)$ which can be represented as a sum of monocomponents the time-frequency representation is composed of distributions of each component (auto-terms) and the interactions of each pair of different components (cross-terms). One way of lowering the
cross-term interferences bases on the convolution the integrand of WVD with smoothing kernel [1,3,8,10]. This direction leads to Cohen's class of transformation with one of the members called Choi-Williams Distribution (CWD) [3,8,10].

Reliability of power electronic drive systems is important in many industrial applications. The analysis of fault mode behavior can be utilized for development of monitoring and diagnostic systems [4,5,6]. In this paper we present also some results of simulation investigations of a converter-fed induction motor drive. PWM converters supplying asynchronous motor were simulated. Detection of irregular frequencies may be useful for diagnosis of some drive faults.

Spectrum of the signal was estimated with the help of the Wavelet Transform (WT) and Wigner-Ville Distribution (WVD) and Choi-Williams Distribution (CWD).

II. WIGNER-VILLE DISTRIBUTION

The Wigner-Ville distribution is time-frequency representation given by [2,8,10]:

\[
W_X(t,\omega) = \int_{-\infty}^{\infty} X(t+\frac{\tau}{2})X^*(t-\frac{\tau}{2})e^{-j\omega \tau}d\tau
\]

where \( t \) is a time variable, \( \omega \) is a frequency variable and \( X(t) \) is analytic form obtained as Hilbert transform on signal \( x(t) \). At least * denotes complex conjugate.

Discrete-time signal \( x(m) \) leads to the discrete time Wigner-Ville distribution. Further, using a sliding symmetrical finite-length analysis window \( h(n) \), the pseudo-Wigner-Ville distribution (PWVD) is obtained:

\[
W_{Xh}(m,\omega) = 2 \sum_{k} X(m+n)X^*(m-n) \times
\begin{align*}
&xh(n)h^*(-n)e^{-j4\pi kn/2L}
\end{align*}
\]

with variables \( k \) corresponds to the discrete frequency.

The Wigner-Ville distribution of a signal can attain negative values. Each time-frequency representation, which preserves marginal conditions cannot be positive everywhere. These local negative values does not have any physical meaning.

One main deficiency of the WVD is the cross-term interference. WVD of the sum of signal components is a linear combination of auto- and cross-terms. Each pair of the signal components creates one additional cross-term in the spectrum, thus confusing the desired time-frequency representation.

III. CHOI-WILLIAMS DISTRIBUTION

Traditionally, the cross-terms are considered as something undesired in the WVD and should be removed. One way of lowering cross-term interference bases on the convolution the integrand of WVD with smoothing kernel [1,3,8,10]. To obtain mentioned goal Choi and Williams introduced the exponential kernel as:

\[
\Phi(\theta,\tau) = e^{-\frac{\theta^2}{2}}
\]

which leads to CWD expressed in equation:

\[
W_{X\Phi}(t,\omega) = \frac{1}{\sqrt{4\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{\sigma^2}{4\pi}} xX^*(u-\frac{\tau}{2})X(u+\frac{\tau}{2})e^{-j\omega \tau}d\sigma du
\]

where \( \sigma (\neq 0) \) is a scaling factor and controls the decay speed of suppressing the cross-terms. The smaller the \( \sigma \) is the more the cross-terms are suppressed. On the other hand, such smoothing reduces the frequency resolution of the WVD and also brings some influence on auto-terms.
IV. WAVELET TRANSFORM

The continuous wavelet transform (CWT) of a signal \( x(t) \) depends on two variables: scale (or frequency) parameter \( a \), and time parameter \( t \). It is given by [9,10]:

\[
\text{CWT}(a,t) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) g\left(\frac{t-t_0}{a}\right) dt
\]

(6)

where \( g(t) \) is the basic (or mother) wavelet, and

\[
\frac{1}{\sqrt{a}} g\left(\frac{t-t_0}{a}\right)
\]

are the wavelet basis functions.

The basic wavelet can be real or complex. The complex Morlet wavelet is defined by:

\[
g(t) = \sqrt{\pi} \cdot f_v \cdot e^{i2\pi f_c t} \cdot e^{-\frac{t^2}{f_v}}
\]

(7)

where: \( f_v \) is a positive bandwidth parameter, \( f_c \) is a wavelet center frequency.

If the sampling period is \( T_s \), it is natural to associate to the scale \( a \) the frequency:

\[
f = \frac{T_s f_c}{a}
\]

(8)

where: \( a \) is a scale, \( T_s \) is the sampling period, \( f_c \) is the center frequency of a wavelet in Hz, \( f \) is the frequency corresponding to the scale \( a \), in Hz.

V. INVESTIGATIONS

This paper shows investigation results for a 3 kVA-PWM-converter with a modulation frequency of 1kHz supplying a 2-pole, 1 kW asynchronous motor (supply voltage 220V, nominal power 1.1 kW, slip 6%, cos \( \alpha = 0.81 \)). (Fig. 1).

Characteristic RC-damping components at the rectifier bridge and at the converter valves are considered. To design the intermediate circuit, the L, C values of a typical 3 kVA converter are chosen.

Fault operation of the inverter drive was considered as short-circuit between two phases with fault resistance 100 \( \Omega \) which occurs at the time 0.1s (Fig. 2).

The signal spectrum has been calculated using the Wavelet Transform (Figs. 3, 4, 5) and the Wigner-Ville Distribution (Figs. 6, 7, 8). The WT enables to detect, before the short circuit, the modulation frequency of the inverter equal to 1000 Hz. When a short circuit occurs, we can see two modulation components (880Hz and 1100Hz) which bring another cross-term component (1060Hz, 940Hz, 530Hz, 470Hz).

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Fig. 6. Time-Frequency representation of the signal taken from the fault operation of the inverter drive, obtained using the Wigner-Ville distribution.

Fig. 7. Cross-section of the time-frequency representation from Fig. 6 for the time \( t = 0.08 \) s.

Fig. 8. Cross-section of the time-frequency representation from Fig. 6 for the time \( t = 0.2 \) s.

The cross-term interference components appear at frequencies which lie between the frequencies of two components. The amplitude of these components is oscillating.

Fig. 9. Time-Frequency representation of the signal taken from the fault operation of the inverter drive, obtained using the Choi-Williams distribution with \( \sigma = 0.05 \).

Fig. 10. Time-Frequency representation of the signal taken from the fault operation of the inverter drive, obtained using the Choi-Williams distribution with \( \sigma = 0.1 \).

When CWD was applied with factor \( \sigma = 0.05 \) suppression effect of the cross-terms can be observed (Fig. 9). For comparison CWD with factor \( \sigma = 0.1 \) was shown in Fig. 10.

VI. CONCLUSIONS

Modern frequency power converters generate a wide spectrum of harmonic components. These are not always revealed by a standard tools of harmonic analysis based on the Fourier transform. Transients resulting from the fault operation of the frequency converter in electrical distribution systems affect power quality. The parameters of transient components have been analyzed using the Wigner-Ville distribution and the wavelets transform. The modern methods enable to detect irregular signal components. This may be useful for the diagnosis of some drive faults.
The Wigner-Ville spectrum of signals with time limited windows shows better frequency concentration and less phase dependence than Fourier spectra. Unfortunately, in the case of multi-component signals (frequency converter), due to the appearance of cross-terms, obtained representation causes sometimes difficulties in fast interpreting. Moreover, the highest frequency component must be less or equal to quarter of multi-component signals (frequency converter), due to windows shows and less phase near the auto-terms, and also the frequency resolution such difficulties in fast interpreting. Moreover, the sampling rate have to be double. AutomaticS. Computer Science and Electronics. University of Mining interferences is to apply a smoothing kernel to the WVD, always remove all artefacts, especially when they appears near the auto-terms, and also the frequency resolution is reduced.

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